

Advanced Structured Materials

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Series Editors

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Common engineering materials are reaching their limits in many applications, and new developments are required to meet the increasing demands on engineering materials. The performance of materials can be improved by combining different materials to achieve better properties than with a single constituent, or by shaping the material or constituents into a specific structure. The interaction between material and structure can occur at different length scales, such as the micro, meso, or macro scale, and offers potential applications in very different fields.

This book series addresses the fundamental relationships between materials and their structure on overall properties (e.g., mechanical, thermal, chemical, electrical, or magnetic properties, etc.). Experimental data and procedures are presented, as well as methods for modeling structures and materials using numerical and analytical approaches. In addition, the series shows how these materials engineering and design processes are implemented and how new technologies can be used to optimize materials and processes.

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Editors

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*Dedicated to our colleague, teacher and
friend Professor Leonid Mikhailovich Zubov
on the occasion of his 80th birthday.*

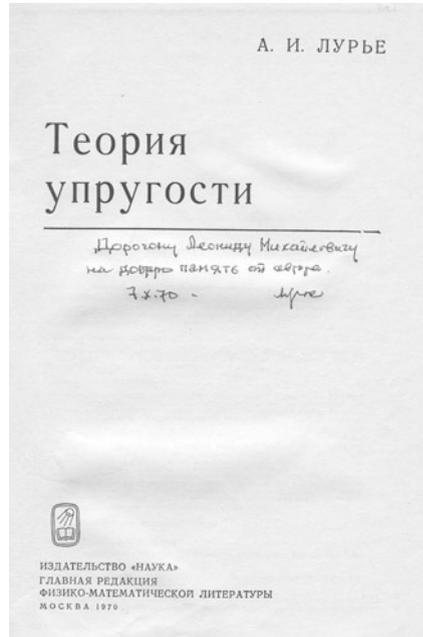
Preface



Leonid M. Zubov. (Photo by Alexey M. Kolesnikov)

Leonid Mikhailovich Zubov was born on September 1, 1943, in Yarensk, a small town near Arkhangelsk, Soviet Union. He graduated from the faculty of physics and mechanics of Leningrad Polytechnical Institute (now Peter the Great St. Petersburg Polytechnic University or simply Polytech). Later, Zubov became a Ph.D. student under supervision of Prof. Anatoliy Lurie. In 1970, Zubov defended his thesis entitled “*Bifurcation of Equilibrium of a Nonlinear Elastic Solid*” at Polytech. It is rather interesting that at that time it was not so common to publish papers with supervisors, so Zubov has no joint publications with Lurie. He published only single-authored papers related to elastic stability and to variational principles in the nonlinear elasticity. On the other hand, he was one of the first readers of Lurie’s book “Theory of elasticity” published in 1970. Moreover, some of his results were included in the

Fig. 1 Title page of Lurie’s “Theory of elasticity” with the dedication to Zubov by the author



book. This fact is mentioned in the book, see also the Lurie’s dedication in Fig. 1. Let us note that his paper on complementary energy in nonlinear elasticity Zubov (1970) found a response in the literature, see, e.g., de Veubeke (1972); Christoffersen (1973); Koiter (1973); Wempner (1980).

In 1970, Zubov moved to Rostov State University, Rostov on Don, in the department of elasticity chaired by Prof. Iosif Vorovich. That time it was not possible for him to stay in Leningrad. So it was a possibility to continue his research in a new department. That time, it was not so easy to find a position in a good university. The letter of recommendation from Prof. Lurie to Prof. Vorovich is given in Fig. 2 (in Russian), where Lurie underlined Zubov’s scientific and personal qualification. In Rostov on Don Zubov continued his research in nonlinear elasticity, theory of shells and started a new topic related to isolated and continuously distributed dislocations and disclinations in solids. In 1986, Zubov defended his doctoral thesis (habilitation), entitled *Semi-inverse and variational methods in nonlinear elasticity*, at Leningrad State University (now St. Petersburg State University).

Working at Rostov State University (now Southern Federal University) Leonid Zubov founded his school in nonlinear elasticity including simple and polar materials, nonlinear theory of shells. He has more than 20 Ph.D. students and two habilitated doctors, see Eremeyev et al. (2014) for more details. He has published more than 120 papers in peer-reviewed journals and several monographs (Zubov, 1982, 1997; Zubov and Karyakin, 2006; Eremeyev and Zubov, 2008, 2009; Zubov and Rudev, 2015).

Глубокоуважаемый Иосиф Израилевич,

обращаюсь к Вам со следующей просьбой. Этой осенью кончат аспирантуру по моей кафедре ("Механика и процессы управления" ЛПИ) Леонид Михайлович Зубов – Вы имели случай его видеть в Агверане. Это в высшей степени способный и трудолюбивый молодой человек. Занимается сейчас задачей устойчивости упругого тела, при чем исходит из строгих уравнений нелинейной теории. Диссертация будет закончена осенью. Частично о его достижениях в этом направлении я упоминал в своем докладе в Агверане. Могу еще добавить, что Л.М.Зубов очень эрудирован во всех разделах механики сплошной среды. Наконец, можно поручиться за его безупречные человеческие качества.

К сожалению, у меня нет возможности сохранить Л.М.Зубова у нас на кафедре – он не ленинградец и нет способа добиться прописки. Циническое разрешение этого вопроса (жениться на жилплощади) Л.М.Зубов отвергает.

Будет очень плохо, если по распределению он попадет на пустое место; поскольку у нас нет лучшего работающего научного коллектива, чем Ваша кафедра в Ростовском университете, я и беру на себя смелость предложить принять на работу Л.М.Зубова. Если Вы найдете это возможным и получите одобрение ректора, то просьба прислать соответствующее письмо, адресовав его ректору Ленинградского ордена Ленина Политехнического института им.М.И.Калинина члену-корреспонденту АН СССР В.С.Смирнову, копию – мне. Комиссия по распределению ожидается в апреле.

Ваш *Лurie*

10.III.1969 г.

Fig. 2 Letter of recommendation from Prof Anatolij Lurie to Prof. Iiosif Vorovich

Last years Prof. Zubov provided research in the following topics:

- Continuum theory of dislocations;
- Mechanics of micropolar solids under finite deformations;
- Nonlinear elasticity for solids with prestressed parts;

(see Zubov and Karyakin (2022a,b, 2023); Goloveshkina and Zubov (2019, 2020, 2021); Zubov (2019, 2021); Zingerman et al. (2023)) and the references therein for previous publications in the field.

This volume of the Advanced Structured Materials Series is devoted to current research in continuum and structural mechanics. It is dedicated to our friend and colleague, Prof. Leonid M. Zubov in occasion of his 80th birthday. With great pleasure that we—his colleagues and Friends—wish Professor Leonid M. Zubov many more creative years of interesting and important research.

Magdeburg, Germany
Cagliari, Italy
July 2023

Holm Altenbach
Victor Eremeyev

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